Construction of a Limaçon as the boundary of an envelope:

- Imagine a stationary circle C1, with a point P riding around its circumference.
- Now imagine that point P is the center of a dynamic circle whose radius is defined by P and a second, stationary point Q.
- The envelope generated by the circles as P moves around C1 is a limaçon\(^1\).
- If the point Q is on circle C1, the boundary of the envelope forms a cardioid\(^2\).
- If the point Q is outside of circle C1, the boundary of the envelope forms a convex limaçon.
- If the point Q is inside of circle C1, the boundary of the envelope forms a dimpled limaçon.

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1 The limaçon of Pascal was first investigated by Dürer, who gave a method for drawing it in Underweysung der Messung (1525). It was rediscovered by Étienne Pascal, father of Blaise Pascal, and named by Gilles-Personne Roberval in 1650 (MacTutor Archive).

2 The name cardioid was first used by de Castillon in Philosophical Transactions of the Royal Society in 1741.
Alternate Construction of a Cardioid as a locus:

- Imagine two tangent circles: a stationary circle $C_1$, with another circle $C_2$ rolling around the circumference.

- Now imagine a point $P$ on the rolling circle. The path described by $P$ as $C_2$ rolls around $C_1$ is an **epicycloid**.

  - If the radius of $C_1$ is the same as the radius of $C_2$ then the epicycloids is a **cardioid**.
Family of Cardioids

Edit the function below to try your own. You can use parameters $a$ and $b$ in the function you create.

$$f(\theta) = a \pm a \cos(\theta)$$

$a = 2.00$

Family of Cardioids

Edit the function below to try your own. You can use parameters $a$ and $b$ in the function you create.

$$f(\theta) = -a \pm a \cos(\theta)$$

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Adapted from *Exploring Precalculus with The Geometer’s Sketchpad 4.0*
http://www.keypress.com/catalog/products/software/Prod_GSPModExpPrecalc.html

Using Geometer’s Sketchpad to Support Mathematical Thinking